

# **Section R.4**

# **Polynomials**

# OBJECTIVE 1

- ✓ **Recognize Monomials**

A **monomial** in one variable is the product of a constant times a variable raised to a nonnegative integer power. A monomial is of the form


$$ax^k$$

where  $a$  is a constant,  $x$  is a variable, and  $k \geq 0$  is an integer. The constant  $a$  is called the **coefficient** of the monomial. If  $a \neq 0$ , then  $k$  is called the **degree** of the monomial.

**EXAMPLE****Examples of Monomials**

<b>Monomial</b>	<b>Coefficient</b>	<b>Degree</b>	
(a) $6x^2$	6	2	
(b) $-\sqrt{2}x^3$	$-\sqrt{2}$	3	
(c) 3	3	0	Since $3 = 3 \cdot 1 = 3x^0, x \neq 0$
(d) $-5x$	-5	1	Since $-5x = -5x^1$
(e) $x^4$	1	4	Since $x^4 = 1 \cdot x^4$

**EXAMPLE****Examples of Nonmonomial Expressions**

- (a)  $3x^{1/2}$  is not a monomial, since the exponent of the variable  $x$  is  $\frac{1}{2}$  and  $\frac{1}{2}$  is not a nonnegative integer.
- (b)  $4x^{-3}$  is not a monomial, since the exponent of the variable  $x$  is  $-3$  and  $-3$  is not a nonnegative integer. 

# OBJECTIVE 2

- ✓ 2 Recognize Polynomials

A **polynomial** in one variable is an algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where  $a_n, a_{n-1}, \dots, a_1, a_0$  are constants,\* called the **coefficients** of the polynomial,  $n \geq 0$  is an integer, and  $x$  is a variable. If  $a_n \neq 0$ , it is called the **leading coefficient**, and  $n$  is called the **degree** of the polynomial.

## EXAMPLE

# Examples of Polynomials

<b>Polynomial</b>	<b>Coefficients</b>	<b>Degree</b>
$-8x^3 + 4x^2 - 6x + 2$	$-8, 4, -6, 2$	3
$3x^2 - 5 = 3x^2 + 0 \cdot x + (-5)$	$3, 0, -5$	2
$8 - 2x + x^2 = 1 \cdot x^2 + (-2)x + 8$	$1, -2, 8$	2
$5x + \sqrt{2} = 5x^1 + \sqrt{2}$	$5, \sqrt{2}$	1
$3 = 3 \cdot 1 = 3 \cdot x^0$	3	0
0	0	No degree

# OBJECTIVE 3

- 3 ✓ Add and Subtract Polynomials

**EXAMPLE****Adding Polynomials**

Find the sum of the polynomials:

$$5x^4 - 3x^3 + 2x - 7 \quad \text{and} \quad -3x^4 + 4x^3 + 7x^2 - 3x + 1$$

**EXAMPLE****Subtracting Polynomials**

Find the difference:

$$(5x^4 - 3x^3 + 2x - 7) - (-3x^4 + 4x^3 + 7x^2 - 3x + 1)$$

# OBJECTIVE 4

✓ **4 Multiply Polynomials**

## EXAMPLE

# Multiplying Polynomials

Find the product:  $(3x - 4)(2x^2 - 4x + 5)$

# OBJECTIVE 5

- ✓ **5 Know Formulas for Special Products**

# FOIL

$$\begin{aligned} (ax + b)(cx + d) &= ax(cx + d) + b(cx + d) \\ &= \underbrace{ax \cdot cx}_{\text{First}} + \underbrace{ax \cdot d}_{\text{Outer}} + \underbrace{b \cdot cx}_{\text{Inner}} + \underbrace{b \cdot d}_{\text{Last}} \\ &= acx^2 + adx + bcx + bd \\ &= acx^2 + (ad + bc)x + bd \end{aligned}$$

**EXAMPLE****Multiplying Polynomials**

(a)  $(x - 4)(x + 4)$

(b)  $(x - 5)^2$

(c)  $(x + 4)(x - 3)$

(d)  $(3x - 4)(5x + 2)$

## Difference of Two Squares

$$(x - a)(x + a) = x^2 - a^2$$

## Squares of Binomials, or Perfect Squares

$$(x + a)^2 = x^2 + 2ax + a^2$$

$$(x - a)^2 = x^2 - 2ax + a^2$$

**EXAMPLE****Using Special Product Formulas**

(a)  $(x - 8)(x + 8)$

(b)  $(x + 3)^2$

(c)  $(4x + 1)^2$

(d)  $(2x - 7)^2$

**EXAMPLE****Cubing a Binomial**

(a)  $(x + 4)^3$

(b)  $(x - 5)^3$

**Cubes of Binomials, or Perfect Cubes**

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$$

## **EXAMPLE** Forming the Difference of Two Cubes

$$\begin{aligned}(x - 1)(x^2 + x + 1) &= x(x^2 + x + 1) - 1(x^2 + x + 1) \\ &= x^3 + x^2 + x - x^2 - x - 1 \\ &= x^3 - 1\end{aligned}$$

## **EXAMPLE** Forming the Sum of Two Cubes

$$\begin{aligned}(x + 2)(x^2 - 2x + 4) &= x(x^2 - 2x + 4) + 2(x^2 - 2x + 4) \\ &= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8 \\ &= x^3 + 8\end{aligned}$$

### **Difference of Two Cubes**

$$(x - a)(x^2 + ax + a^2) = x^3 - a^3$$

### **Sum of Two Cubes**

$$(x + a)(x^2 - ax + a^2) = x^3 + a^3$$

# OBJECTIVE 6

- ✓ **6 Divide Polynomials Using Long Division**

**EXAMPLE****Dividing Two Integers**

Divide 987 by 14

$$(\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}$$

**EXAMPLE****Dividing Two Polynomials**

Find the quotient and the remainder when

$$2x^3 - 3x^2 + 5x + 2 \text{ is divided by } x^2 + 1$$

**EXAMPLE****Dividing Two Polynomials**

Find the quotient and the remainder when

$$x^4 - 3x^3 + 2x - 5 \text{ is divided by } x^2 + x + 1$$

# THEOREM

Let  $Q$  be a polynomial of positive degree and let  $P$  be a polynomial whose degree is greater than or equal to the degree of  $Q$ . The remainder after dividing  $P$  by  $Q$  is either the zero polynomial or a polynomial whose degree is less than the degree of the divisor  $Q$ .

# OBJECTIVE 7

- ✓ **Work with Polynomials in Two Variables**

## EXAMPLE

### Examples of Polynomials in Two Variables

$$3x^2 + 2x^3y + 5$$

Two variables,  
degree is 4.

$$\pi x^3 - y^2$$

Two variables,  
degree is 3.

$$x^4 + 4x^3y - xy^3 + y^4$$

Two variables,  
degree is 4.

**EXAMPLE****Using a Special Product Formula**

Use the Square of a Binomial formula to multiply

$$(3x - 2y)^2$$