

Symmetry; Key Equations

2.1

Testing for symmetry

- A graph is symmetric with respect to the x -axis if for all (x,y) on the graph $(x,-y)$ is also on the graph. Replace y with $-y$.
- A graph is symmetric with the y -axis if for all (x,y) on the graph $(-x, y)$ is also on the graph. Replace x with $-x$.
- A graph is symmetric with the origin if for all (x,y) on the graph $(-x,-y)$ is also on the graph. Replace x with $-x$ and y with $-y$.

Testing for symmetry

- In a graph, symmetry can easily be determined by “folding” the graph over the x-axis, y-axis, or the origin.
- In an equation, symmetry is obtained by replacing x , y , or both with their opposites and observing if equivalent equations result. The new equation must match the original exactly.

Test the following equations for symmetry, then find the intercepts.

$$\text{EX: } y = 4x^2 + x - 2$$

$$\text{EX: } x^2 + (y - 2)^2 = 4$$

$$\text{EX: } y^2 = 1 - x^2$$

$$\text{EX: } y = \frac{x^2 - 4}{x^3 + x}$$

$$\text{EX: } y = \frac{x^3 - x}{x^5 + x^3}$$

Key Equations

$$y = x$$

$$\text{Domain} = (-\infty, \infty)$$

$$\text{Range} = (-\infty, \infty)$$

Symmetry : Origin

$$y = x^2$$

$$\text{Domain} = (-\infty, \infty)$$

$$\text{Range} = [0, \infty)$$

Symmetry : Y-axis

Key Equations

$$y = x^3$$

$$\text{Domain} = (-\infty, \infty)$$

$$\text{Range} = (-\infty, \infty)$$

Symmetry : Origin

$$x = y^2$$

$$\text{Domain} = [0, \infty)$$

$$\text{Range} = (-\infty, \infty)$$

Symmetry : X-axis

Key Equations

$$y = \frac{1}{x}$$

$$\text{Domain} = (-\infty, 0) \cup (0, \infty)$$

$$\text{Range} = (-\infty, 0) \cup (0, \infty)$$

Symmetry : Origin